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2.1 Basic laws

2.1.1 Planck law

All objects above the temperature of absolute zero emit thermal radiation due to thermal motion of the atoms and the molecules. The hotter they are, the more they emit. Spectral distribution of thermal radiation emitted by an ideal radiator -a blackbody - is described by the Planck law

$$M_{\lambda}(T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k T} - 1\right)} \tag{2.1}$$

where M_λ is the spectral radiant exitance in W m^{-2} μm^{-1}, T is the blackbody temperature in Kelvins, λ is the wavelength in μm, h=6.626176 $\times$ 10^{-34} J s is thePlanck constant, c = 2.9979246 10^{8} m/s is the speed of the light in the vacuum, k = 1.380662 $\times$ 10^{-23} J K^{-1} is the Boltzmann's constant.
It is common to express the Eq. (2.1) in the form

$$M_{\lambda}(T) = \frac{c_1}{\lambda^5\left(e^{c_2/\lambda T} - 1\right)} \tag{2.2}$$

where c_1 and c_2 are the constants of following values c_1=3.741832 $\times$ 10^{4} [W cm^{-2} μm^{4}] and c_2 = 14387.86 μm K. However, it must be noted that it is possible to find in literature different values of the constants c_1 and c_2 because as values of many others physical constants they are continually refined when improved measurement techniques become available. Additionally, we must remember that the constants c_1 and c_2 can be presented using other units, too.

The spectral radiant exitance M_λ in the form shown in Eq. (2.2) expresses the power of radiation within a spectral interval of 1 μm around the wavelength λ emitted into a hemisphere by a blackbody having an area of 1cm^2.
It is customary to refer all radiometric measurements and calculations to a spectral interval equal to the unit in which wavelength is measured. As wavelength in optical radiometry is usually expressed in μm therefore the spectral interval is expressed in μm, too. However, we must remember that in practical measurements the spectral interval of the measuring device usually differ from 1μm.

The term *blackbody* used above describes a body that allows all incident radiation to pass into it and absorbs internally all of the incident radiant energy. This must be true for all wavelengths and for all angles of incidence. Blackbody is also the most efficient radiator. A perfect blackbody at room temperatures would appear totally black to the eye, hence the origin of the name. The spectral radiant exitance of a blackbody at temperatures ranging from room temperature up to temperature of the Sun is shown in Fig. 2.1.

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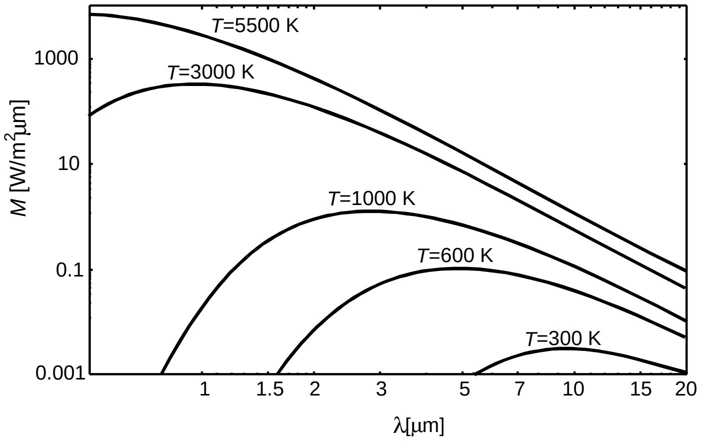

Fig. 2.1. Spectral radiant exitance of a blackbody at different temperatures

The Planck law enables calculation of the spectral radiant exitance M_λ at and is very useful in many radiometric calculations. However, sometimes it can be also interesting to determine the blackbody temperature T when its radiant exitance M_λ is known. It can be done using a following formula that can be treated as an inverse Planck law

$$T = \frac{c_2}{\ln\left[\frac{\left(c_1 + \lambda^5 M_\lambda\right)^\lambda}{\left(\lambda^5 M_\lambda\right)^\lambda}\right]} \tag{2.3}$$

The relationship between the temperature T and the spectral exitance M_λ for different wavelengths λ calculated using the Eq. (2.3) is presented in Fig. 2.2. The Eq. (2.3) can be used for calculation of object temperature when its radiant spectral exitance M_λ for a narrow spectral band is measured.

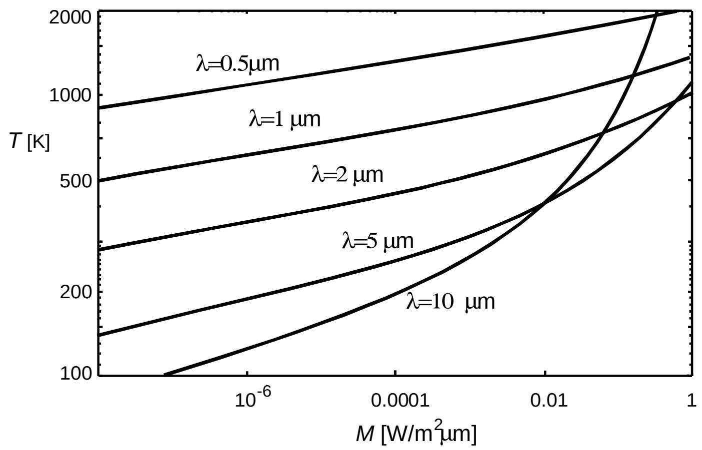

Fig. 2.2. Relationship between the temperature *T* and the radiant spectral exitance M_λ for the different wavelengths λ

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2.1.2 Wien law

The Wien law represents a simplified version of the Planck law on assumption that exp[(c_2/λ T) – 1]≈ [(c_2/λ T)

$$M_{\lambda}(T) = \frac{c_1}{\lambda^5 \exp(c_2/\lambda \mathrm{T})}. \quad (2.4)$$

The relative error of exitance calculation using the Wien law can be defined as ratio of the difference of exitance calculated using the Wien law *M (Wien)* and exitance calculated using the Planck law *M (Planck)* to exitance calculated using the Planck law *M (Planck)*

$$rel\_error = \frac{M_{\lambda}(Wien) - M_{\lambda}(Planck)}{M_{\lambda}(Planck)} \quad (2.5)$$

The values of relative error *rel_error* for different wavelengths and temperatures using the Eq. (2.5) are shown in Fig. 2.3. As it can be seen the *rel_error* rises with temperature and wavelength. If we make some calculations using Eq. (2.5) then we can come to two more details rules. First, that the error caused by using Wien law is about 1.5% for wavelength of maximum radiation and decreases rapidly for shorter wavelengths. Second, when λ T ≤ 3000 μm K then the *rel_error* ≤ 1%.

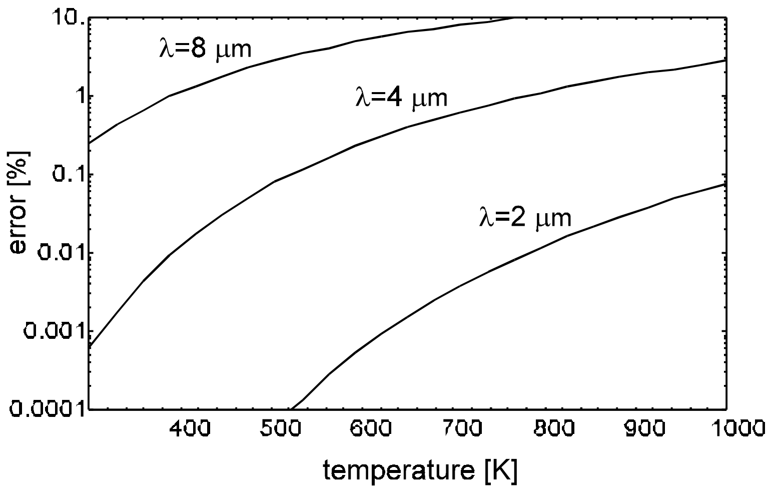

Fig. 2.3. Relative error of exitance calculation using the Wien law

Application of the Wien law causes decrease in calculations accuracy but it also significantly simplifies calculation of many equations with integrals, etc. Therefore, the Wien law was often used instead of the Planck law in the past before introduction of personal computers. Nowadays, commonly available personal computers can solve analytically or numerically many sophisticated formulas within seconds and the Planck law is typically used.

2.1.3 Stefan-Boltzmann law

Integrating Plack's law over wavelength from zero to infinity gives an expression for radiant exitance, the flux radiated into a hemisphere by a blackbody of a unit area. This total radiant flux emitted from the surface of an object at the temperature T is expressed by Stefan-Boltzman law, in the form

$$M = \sigma T^4, \quad (2.6)$$

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where M is the radiant exitance of a blackbody in unit W/m^2 and σ is the Stefan-Boltzman constant. The currently recommended value for σ is 5.67032×10^{-8} W m^{-2} K^{-4}.

Results of calculations of dependence of the radiant exitance M on the blackbody temperature T are shown in Fig. 2.4.

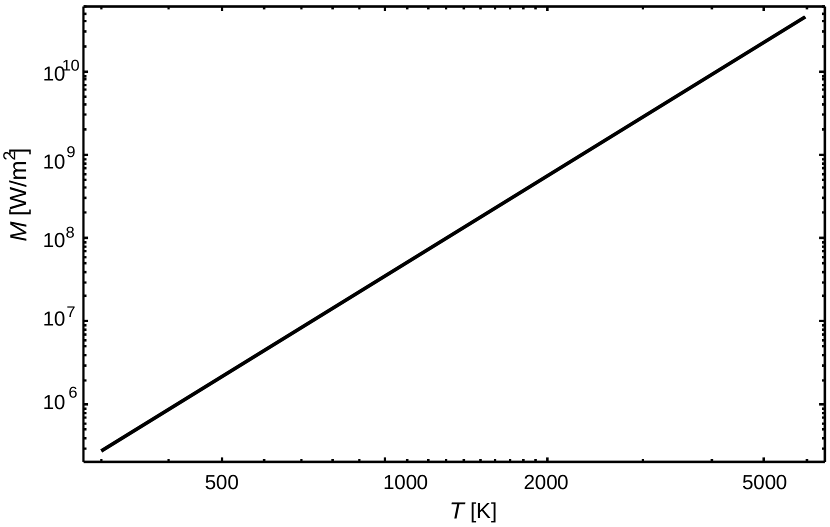

Fig. 2.4. Dependence of radiant exitance *M* on blackbody temperature *T*

2.1.4 Wien displacement law

If one differentiates the Planck formula and solves for the maximum yields a simple relationship between the wavelength λ_{max} where the Planck formula has its maximum value of radiant exitance $M_{\lambda max}$ and the temperature T of the blackbody. The resulting relationship is called Wien's displacement law and is given by

$$\lambda_{max} T = A \tag{2.7}$$

where A=2897.8 in μm K.

Dependence of wavelength at which the maximum spectral exitance occurs on temperature is shown in graphical form in Fig. 2.5. As we can see in this figure, the λ_{max} varies inversely with absolute temperature and for typical temperatures met in industry, science, environment etc. is located within range of infrared radiation. It is the main reason why most of non-contact system for temperature measurement are infrared systems.

Using the Wien displacement law (2.7) and the Planck law (2.2) we can calculate the radiant exitance M_λ at wavelength λ_{max} at which the maximum spectral exitance occurs for any temperature. The dependence of $M(\lambda=\lambda_{max})$ on temperature T is presented in Fig. 2.6.

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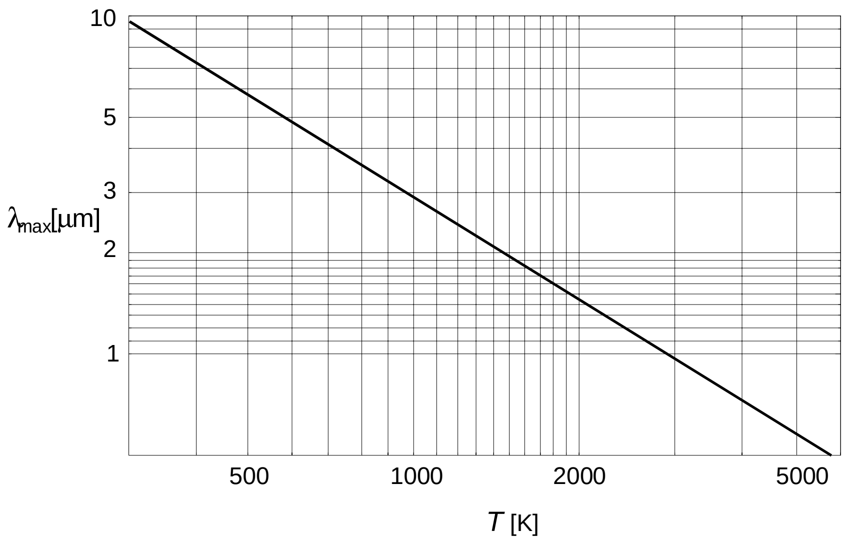

Fig. 2.5. Dependence of wavelength λ_{max} at which the maximum spectral exitance occurs on temperature *T*

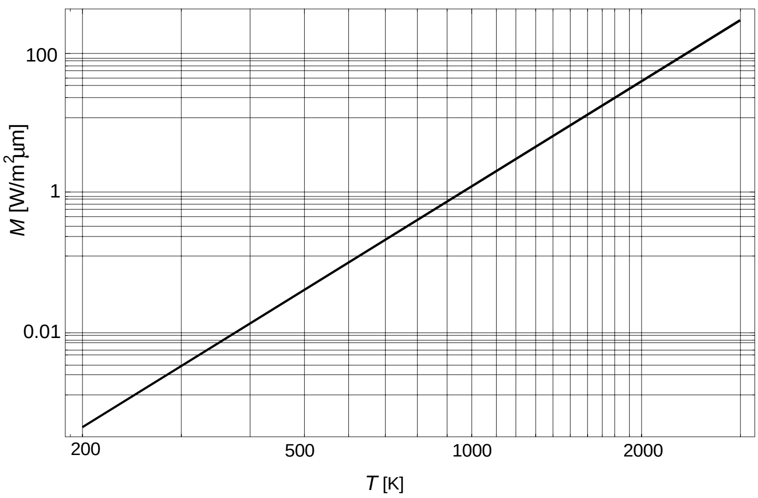

Fig. 2.6. Function of radiant exitance M_λ at wavelengths λ_{max} at which the maximum spectral exitance occurs on temperature *T*

2.1.5 Lambert (cosine) law

When radiance of an element of a surface is the same in all direction within the hemisphere over this element then the following relationship is fulfilled

$$I(\theta) = I_n \cos\theta \tag{2.8}$$

where $I(\theta)$ is the radiant intensity in direction of the angle θ to the direction normal to the surface, I_n is the radiant intensity in the direction normal to the surface.

An ideal surface that fulfils the Lambert cosine law is called the Lambertian surface. For the Lambertian surface there exists a following relationship between the radiant exitance *M* and the radiant radiance *L*

$$M = \pi L \, . \tag{2.9}$$

The relationship (2.9) is sometimes called the Lambert law instead of the earlier presented law.